Quasidifferential Saves Infeasible Differential
Improved Weak-Key Key-Recovery Attacks on Round-Reduced GIFT

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Abstract. GIFT, including GIFT-64 and GIFT-128, is a family of lightweight block ciphers with outstanding implementation performance and high security, which is a popular underlying primitive chosen by many AEADs such as SUNDAE-GIFT. Currently, differential cryptanalysis is the best key-recovery attack on both ciphers, but they have stuck at 21 and 27 rounds for GIFT-64 and GIFT-128, respectively. Recently, Beyne and Rijmen proposed the quasidifferential transition matrix for differential cryptanalysis at CRYPTO 2022 and showed that the fixed-key probability of a differential (characteristic) can be expressed as the sum of correlations of all quasidifferential trails corresponding to this differential (characteristic). As pointed out by Beyne and Rijmen in their paper, the quasidifferential methodology is useful in identifying weak-key differential attacks.

In this paper, we apply Beyne and Rijmen's method to GIFT. Some differential characteristics with small (average) probabilities can have much larger probabilities when weak-key conditions hold. Improved weak-key differential attacks on GIFT-64 and GIFT-128 are thus obtained. For GIFT-64, the probability of a 13-round differential is improved from $2^{-62.06}$ to $2^{-57.82}$ with 4 bits of weak-key conditions, then an improved differential key-recovery attack on 21-round GIFT-64 is obtained with $2^{117.42}/2^{64}$ time/data complexities; the probability of a 13-round multiple differential (containing 33 characteristics) is improved from $2^{-58.96}$ to $2^{-55.67}$ with 4 bits of weak-key conditions, then an improved multiple differential key-recovery attack on 21-round GIFT-64 is obtained with $2^{123.27}/2^{64}$ time/data complexities. For GIFT-128, the probability of a 20-round differential is improved from $2^{-121.83}$ to $2^{-114.77}$ with 6 bits of weak-key conditions; the probability of a 21-round multiple differential (containing 2 differentials) is improved from $2^{-128.38}$ to $2^{-122.77}$ with 4 bits of weak-key conditions. Improved (multiple) differential weak-key key-recovery attacks are obtained for 27 and 28 rounds of GIFT-128 with $2^{115.77}/2^{115.77}$ and $2^{123.77}/2^{123.77}$ time/data complexities, respectively.

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As far as we know, this is the first time that a (weak-key) key-recovery attack can reach 28 rounds of GIFT-128.

Additionally, as an independent interest, we perform the first differential attack on SUNDAE-GIFT. The differential used in this attack is checked with quasidifferential trails, thus the probability is reliable. Our attack is nonce-respecting and has significantly better complexities than the currently best attack.

Keywords: Quasidifferential · Differential · Weak-Key · GIFT

1 Introduction

Differential cryptanalysis is one of the most fundamental methods to analyze the security of block ciphers, which was proposed by Biham and Shamir [6] at CRYPTO 1990 to attack the block cipher DES. The core concept of the differential attack is to construct the probabilistic transition from an input difference to an output difference for iterated ciphers. This involves choosing plaintext pairs that satisfy the input difference, tracing the difference transitions of the iterated block cipher throughout the multi-round encryption process, and finding high-probability differentials over a large number of rounds. The differential transition that traces the large number of rounds is usually converted to tracing intermediate differences, the calculation of the differential probability of long rounds can be estimated heuristically as the product of multiple probabilities of intermediate differences, and Lai et al. [13] showed that it yields the correct value of the *key-averaged probability* for *Markov ciphers*. In addition, Lai et al. introduced an additional assumption, which is called the *hypothesis of stochastic equivalence*, to deal with the problem that the actual probability may be different from *key-averaged probability*, which is caused by the fixed-key throughout a differential attack. This assumption states that the probability for each key is close to the average probability.

At CRYPTO 2022, Beyne and Rijmen [5] proposed the quasidifferential transition matrix in differential cryptanalysis, and established the connection between the correlation matrix in linear cryptanalysis [10] and the quasidifferential transition matrix in differential cryptanalysis. The quasidifferential transition matrix satisfies similar properties to the correlation matrix in linear cryptanalysis. One such property is that the fixed-key probability of a differential can be expressed as the sum of the correlations of all its quasidifferential trails, without any assumption. Given one differential (characteristic), correlations of its corresponding quasifferentials are heavily affected by keys. Thus, differential probability in different key spaces can be analyzed relatively more easily now. It is not surprising that in some key spaces, the differential probability would be significantly larger than others.

GIFT [4] is a lightweight block cipher with two versions: GIFT-64 and GIFT-128. The outstanding implementation performance and high security make GIFT be a popular underlying primitive for many Authenticated Encryptions with

Associated Data (AEADs) such as SUNDAE-GIFT [2], GIFT-COFB [3], and HyENA [8]. In the security evaluation of GIFT, differential cryptanalysis is currently the most effective attack compared with other attacks. At CT-RSA 2019, Zhu et al. [23] proposed the first third-party cryptanalysis on GIFT, which includes a 19-round attack on GIFT-64 and a 22-round attack on GIFT-128, respectively. Sasaki et al. [16] improved the meet-in-the-middle (MitM) attack on 15-round GIFT-64. The 20-round and 21-round differential attacks on GIFT-64 were proposed by Chen et al. [9] at ICISC 2019 using the full codebook. Chen et al. [9] also performed the 20-round differential key-recovery attack on GIFT-64 without the full codebook. Li et al. [14] proposed the 26-round differential attack on GIFT-128, and the 26-round attack is improved by Ji et al. [12] at SAC 2021. At FSE 2021, Zong et al. [25] proposed the key-recovery-attack friendly distinguishers, gave a 27-round differential key-recovery attack and a 22-round linear key-recovery attack on GIFT-128, respectively. For the linear cryptanalysis of GIFT, Sun et al. [19] proposed a 19-round linear attack on GIFT-64 at SAC 2021, and a 24-round linear attack on GIFT-128 at FSE 2021 [20]. After that, Sun et al. [21] gave a 25-round linear attack on GIFT-128 at FSE 2022, and improved the 25-round linear attack to increase the success probability by using more data and higher time complexity. Antonio Flórez-Gutiérrez et al. [11] further improved the complexity and success probability of the 25-round linear key-recovery attack on GIFT-128 at EUROCRYPT 2024. In addition, Wang et al. [22] proposed the differential-linear attacks on 18-round GIFT-64 and on 19-round GIFT-128 at CIC 2024. All the above attacks are under the single-key setting. In the relate-key setting, Liu et al. [15] proposed a 21-round boomerang attack on GIFT-128, and Ji et al. [12] proposed a 23-round rectangle attack on GIFT-128 at SAC 2021.

Although extensive cryptanalysis from the community has been applied to GIFT, the security of GIFT is still strong. All attacks stuck at 21 and 27 rounds for GIFT-64 and GIFT-128 in the single-key setting, respectively. Considering the importance of GIFT (as mentioned, GIFT-128 is the underlying primitive of three NIST LWC candidates), a better evaluation on GIFT's security is always warranted, even in the extreme scenarios such as under the weak-key settings.

On the other hand, Beyne and Rijmen's quasidifferential approach has been successfully applied to RECTANGLE, KNOT, SPECK, and SIMON, and new weak-key attacks are presented. However, few works tried to extend their attacks to more ciphers, to the best of our knowledge. In this paper, we aim to bridge the gap by studying how to utilize the quasidifferential method to analyze the security of GIFT.

Our Contributions. This paper applies Beyne and Rijmen's quasidifferential approach [5] to GIFT, studying the differential attacks in the weak-key setting. Some differential characteristics with small probabilities that are infeasible in a normal differential attack can be used now in a weak-key setting, by putting some weak-key conditions. The best attacks on GIFT-64 and GIFT-128 are provided, in terms of the complexity or the number of rounds.

Concretely, by diving deep into the linear key-schedule of GIFT, we extract linear equations for round-key bits, based on the signs of correlations of quasid-

Table 1. Summary of the attack results on GIFT-64, GIFT-128 and SUNDAE-GIFT, respectively. SK stands for single-key setting. RK stands for related-key setting.

Algorithm	Attack Type	Rounds	Scenario	# Keys	Time	Data	Memory	P_S	Reference
GIFT-64	Boomerang	23	RK‡	2^{128}	$2^{126.60}$	$2^{63.30}$	-	-	[15]
	Rectangle	24	RK‡	2^{128}	$2^{106.00}$	$2^{63.78}$	$2^{64.10}$	-	[12]
	Rectangle	25	RK‡	2^{128}	$2^{120.92}$	$2^{63.78}$	$2^{64.10}$	-	[12]
	Differential	26	RK‡	2^{128}	$2^{123.23}$	$2^{60.96}$	$2^{102.86}$	-	[18]
	Differential	26	RK‡	2^{128}	$2^{115.96}$	$2^{60.96}$	$2^{102.86}$	-	[7]
	Differential-Linear	18	SK	2^{128}	$2^{124.61}$	$2^{61.57}$	-	-	[22]
	Linear	19	SK	2^{128}	$2^{127.11}$	$2^{62.96}$	2^{60}	60%	[19]
	Differential	19	SK	2^{128}	2^{112}	2^{63}	2^{80}	-	[23]
	Multiple Differential	20	SK	2^{128}	$2^{112.68}$	2^{62}	2^{112}	-	[9]
	Differential	20	SK	2^{128}	$2^{101.68}$	2^{64}	2^{96}	-	[9]
	Multiple Differential	**21**	**SK**	$\mathbf{2^{124}}$	$\mathbf{2^{123.27}}$	$\mathbf{2^{64}}$	$\mathbf{2^{112}}$	**99.9%**	**Sect. 5.1**
	Differential	21	SK	2^{128}	$2^{121.66}$†	2^{64}	2^{96}	51.60%†	[9]
	Differential	**21**	**SK**	$\mathbf{2^{124}}$	$\mathbf{2^{117.42}}$	$\mathbf{2^{64}}$	$\mathbf{2^{96}}$	**81.06%**	**Sect. 5.1**
	Differential	**21**	**SK**	$\mathbf{2^{124}}$	$\mathbf{2^{120.60}}$	$\mathbf{2^{64}}$	$\mathbf{2^{96}}$	**99.41%**	**Sect. 5.1**
GIFT-128	Boomerang	21	RK‡	2^{128}	$2^{126.6}$	$2^{126.6}$	$2^{126.6}$	-	[15]
	Rectangle	23	RK‡	2^{128}	$2^{126.89}$	$2^{121.31}$	$2^{121.63}$	-	[12]
	Differential-Linear	19	SK	2^{128}	$2^{121.53}$	$2^{122.51}$	-	-	[22]
	Differential	22	SK	2^{128}	2^{120}	2^{120}	2^{86}	-	[24]
	Linear	22	SK	2^{128}	2^{117}	2^{117}	2^{78}	-	[25]
	Linear	24	SK	2^{128}	$2^{124.45}$	$2^{122.55}$	2^{105}	80.01%	[20]
	Linear	25	SK	2^{128}	$2^{126.77}$	$2^{124.75}$	2^{96}	50%	[21]
	Linear	25	SK	2^{128}	$2^{127.77}$	$2^{125.75}$	2^{96}	75%	[21]
	Linear	25	SK	2^{128}	$2^{124.61}$	$2^{123.02}$	2^{112}	80%	[11]
	Differential	26	SK	2^{128}	$2^{124.415}$	2^{109}	$2^{124.415}$	-	[14]
	Differential	26	SK	2^{128}	$2^{123.245}$	$2^{123.245}$	2^{109}	-	[12]
	Differential	27	SK	2^{128}	$2^{124.83}$	$2^{123.53}$	2^{80}	-	[25]
	Differential	**27**	**SK**	$\mathbf{2^{122}}$	$\mathbf{2^{115.77}}$	$\mathbf{2^{115.77}}$	$\mathbf{2^{92}}$	**99.9%**	**Sect. 5.2**
	Multiple Differential	**28**	**SK**	$\mathbf{2^{124}}$	$\mathbf{2^{123.77}}$	$\mathbf{2^{123.77}}$	$\mathbf{2^{96}}$	**86.5%**	**Sect. 5.2**
SUNDAE-GIFT	Linear	16	Nonce-respecting	2^{128}	$2^{91.20}$	$2^{60.00}$	2^{96}	-	[25]
	Linear	17	Nonce-respecting	2^{128}	$2^{123.38}$	$2^{61.51}$	2^{49}	80.01%	[20]
	Differential	**17**	**Nonce-respecting**	$\mathbf{2^{128}}$	$\mathbf{2^{75.37}}$	$\mathbf{2^{62.5}}$	$\mathbf{2^{66}}$	**99.9%**	**Sect. 6**

† The 21-round differential key-recovery attack on GIFT-64 presented by [9] with $2^{107.61}/2^{64}/2^{96}$ time/data/memory complexities, which is the combination of the 1-round attack and the 20-round attack, is not accurate, we reevaluate the complexities and success probability by the successive 21-round attack.

‡ Note that there is no security claim of GIFT under the related-key setting, the results under the related-key setting are shown in grey.

ifferential trails, and convert them to equations for master-key bits. Then the weak-key space of master-key bits is obtained, and the probability of the differential characteristic in this weak-key space is improved. Furthermore, in order to use the *differential* to amplify the probability, we introduce a method to derive the best weak-key conditions from all the characteristics of the differential. These analyses are based on a reasonable assumption that the exact probability of a differential characteristic can be approximated by the sum of correlations of those trails whose absolute correlation is equal to the average probability of the characteristic. We have done experiments on GIFT-64 and GIFT-128 to verify the validity of the assumption, and the experimental results are exactly consistent with our assumption.

After applying the quasidifferential cryptanalysis to GIFT, we present improved (multiple) differential attacks on GIFT-64 and GIFT-128 in the weak-key settings, respectively. For GIFT-64, the probability of a 13-round differential is improved from $2^{-62.06}$ to $2^{-57.82}$ with 4-bit conditions of master-key, which can mount a 21-round differential attack with $2^{117.42}/2^{64}$ time/data complexities. The probability of a 13-round multiple differential of GIFT-64 is improved from

$2^{-58.96}$ to $2^{-55.67}$ with 4-bit conditions of master-key, which can boost the multiple differential attack on GIFT-64 from 20-round to 21-round with $2^{123.27}/2^{64}$ time/data complexities. For GIFT-128, the probability of a 20-round differential is improved from $2^{-121.83}$ to $2^{-114.77}$ with 6-bit conditions of master-key, and the probability of a 21-round multiple differential is improved from $2^{-128.38}$ to $2^{-122.77}$ with 4-bit conditions of master-key. The 27-round and the first 28-round improved (multiple) differential attacks on GIFT-128 are obtained with $2^{115.77}/2^{115.77}$ and $2^{123.77}/2^{123.77}$ time/data complexities, respectively.

Finally, an 11-round differential characteristic whose probability is 2^{-60} without conditions of master-key can be utilized to launch the first differential attack on 17-round SUNDAE-GIFT with $2^{75.37}/2^{62.5}$ time/data complexities. The related results and our attacks are summarized in Table 1.

The source code, the results of this paper, and the full paper are provided at https://github.com/ccc53021/quasidifferential-gift.

Outline. In Sect. 2, we briefly describe the differential cryptanalysis, linear cryptanalysis, and quasidifferential proposed in [5], define some notations, recall the description of GIFT. In Sect. 3, we revisit and discuss the roles of the keys in the exact probability of a characteristic. In Sect. 4, we introduce how to apply the quasidifferential cryptanalysis to GIFT, derive weak-key differential (characteristic) distinguishers, and make some experiments on GIFT. Section 5 presents the weak-key (multiple) differential attacks on GIFT-64 and GIFT-128, respectively, and the attack on SUNDAE-GIFT is given in Sect. 6. Section 7 concludes this paper.

2 Preliminaries and Related Works

In this section, we first recall the differential cryptanalysis and linear cryptanalysis, and introduce the quasidifferential proposed by Beyne and Rijmen [5], which shows that the fixed-key probability of a differential can be expressed as the sum of the correlations of its quasidifferential trails. In addition, we define the notations used in this paper and briefly review the description of GIFT and SUNDAE-GIFT with GIFT-128 as an underlying primitive.

2.1 Review of Differential Cryptanalysis in Fixed-Key Model

Differential Cryptanalysis. Differential cryptanalysis [6] is used to analyze the propagation of differences through the function $\mathtt{F} : \mathbb{F}_2^n \to \mathbb{F}_2^m$, and find a differential with high probability to attack the cipher. The target of the adversary is to find a differential $(a, b) \in \mathbb{F}_2^n \times \mathbb{F}_2^m$ that maximizes the number of solutions x to the difference equation

$$\mathtt{F}(x \oplus a) \oplus \mathtt{F}(x) = b. \tag{1}$$

The difference distribution table of $\mathtt{F}$, denoted as $\mathtt{DDT}^{\mathtt{F}}$, is a $2^n \times 2^m$ table with rows and columns indexed by input and output differences. The corresponding entries are equal to the number of solutions x for a particular differential (a, b):

$$\mathtt{DDT}^{\mathtt{F}}_{(a,b)} = \left|\{x \in \mathbb{F}_2^n | \mathtt{F}(x \oplus a) \oplus \mathtt{F}(x) = b\}\right|. \tag{2}$$

Differential cryptanalysis typically focuses on functions F structured as compositions $\mathsf{F} = \mathsf{F}_r \circ \mathsf{F}_{r-1} \circ \cdots \circ \mathsf{F}_1$, where individual functions F_i exhibit differentials with relatively high probability, making them more suitable for analysis. Thus, it is possible to estimate the probability of differential (a_1, a_{r+1}) based on *characteristics*. A characteristic is a sequence $(a_1, a_2, \ldots, a_{r+1})$ of compatible differences between intermediate inputs and outputs through each F_i. The estimation of characteristic probabilities frequently relies on the assumption of independence among intermediate differentials:

$$\Pr[\wedge_{i=1}^{r} \mathsf{F}_i(x_i \oplus a_i) \oplus \mathsf{F}_i(x_i) = a_{i+1}] \approx \prod_{i=1}^{r} \Pr[\mathsf{F}_i(x_i \oplus a_i) \oplus \mathsf{F}_i(x_i) = a_{i+1}]. \quad (3)$$

When functions $\mathsf{F}_1, \ldots, \mathsf{F}_r$ are dependent on keys $k_1, \ldots, k_r$, the heuristic proposed in Eq. (3) can be justified by applying the *Markov cipher* assumption [13]. Specifically, it has been demonstrated that if all round keys are uniformly random and independent, the *key-averaged probability* of a characteristic aligns with the product of intermediate key-averaged probabilities.

Quasidifferential. Beyne and Rijmen [5] proposed the quasidifferential framework by introducing the quasidifferential transition matrices as a differential analog of correlation matrices [10] to achieve a more complete understanding of differential cryptanalysis.

Definition 1 (Quasidifferential basis [5]). *Let n be a positive integer. For any $u, a \in \mathbb{F}_2^n$, the function $\beta_{u,a} : \mathbb{F}_2^n \times \mathbb{F}_2^n \to \mathbb{R}$ is defined by*

$$\beta_{u,a}(x, y) = \chi_u(x)\delta_a(x + y). \quad (4)$$

The set of all $\beta_{u,a}$ is called the quasidifferential basis for $\mathbb{R}[\mathbb{F}_2^n \times \mathbb{F}_2^n]$.

The functions δ_x such that $\delta_x(y) = 1$ if $y = x$ and zero elsewhere, and the functions χ_x such that $\chi_u(x) = (-1)^{u^{\mathsf{T}}x}$ with $u \in \mathbb{F}_2^n$. The functions $\beta_{u,a}$ are not only linearly independent but also orthogonal. Similar to the Fourier transformation, Beyne and Rijmen define the change-of-basis operator $\mathscr{Q}_n : \mathbb{R}[\mathbb{F}_2^n \times \mathbb{F}_2^n] \to \mathbb{R}[\mathbb{F}_2^n \times \mathbb{F}_2^n]$ by $(\mathscr{Q}_n f)(u, a) = \langle \beta_{u,a}, f \rangle$. The definition of the *quasidifferential transition matrix* using the change-of-basis operator $\mathscr{Q}_n$ and the transition matrix for pairs of values is in Definition 2. The Kronecker (or tensor) product $T^{\mathsf{F}} \otimes T^{\mathsf{F}}$ is defined as a $2^{2m} \times 2^{2n}$ matrix with coordinates

$$(T^{\mathsf{F}} \otimes T^{\mathsf{F}})_{(y_1,y_2),(x_1,x_2)} = T^{\mathsf{F}}_{y_1,x_1} T^{\mathsf{F}}_{y_2,x_2} = \delta_{y_1}(\mathsf{F}(x_1))\delta_{y_2}(\mathsf{F}(x_2)). \quad (5)$$

Definition 2 (Quasidifferential transition matrix [5]). *Let n and m be two positive integers and $\mathsf{F} : \mathbb{F}_2^n \to \mathbb{F}_2^m$ a function. The quasidifferential transition matrix D^{F} is defined as the matrix-representation of $T^{\mathsf{F}} \otimes T^{\mathsf{F}}$ with respect to the quasidifferential basis defined in Definition 1. That is, $D^{\mathsf{F}} = \mathscr{Q}_m(T^{\mathsf{F}} \otimes T^{\mathsf{F}})\mathscr{Q}_n^{-1}$.*

As described in [5], the coordinates of D^{F} are pairs $(u, a) \in \mathbb{F}_2^n \times \mathbb{F}_2^n$ and $(v, b) \in \mathbb{F}_2^m \times \mathbb{F}_2^m$. By the orthogonality of the quasidifferential basis, it holds

that $\mathscr{Q}_n^{-1} = \mathscr{Q}_n^{\mathsf{T}}/2^n$ and consequently

$$\begin{aligned} D^{\mathsf{F}}_{(v,b),(u,a)} &= \langle \delta_{(v,b)}, \mathscr{Q}_n(T^{\mathsf{F}} \otimes T^{\mathsf{F}})\mathscr{Q}_n^{\mathsf{T}}\delta_{(u,a)}\rangle/2^n = \langle \beta_{v,b}, (T^{\mathsf{F}} \otimes T^{\mathsf{F}})\beta_{u,a}\rangle/2^n \\ &= \frac{1}{2^n} \sum_{(x,y)\in\mathbb{F}_2^n\times\mathbb{F}_2^n} \chi_u(x)\chi_v(\mathsf{F}(x))\delta_a(x+y)\delta_b(\mathsf{F}(x)+\mathsf{F}(y)) \\ &= \frac{1}{2^n} \sum_{\substack{x\in\mathbb{F}_2^n \\ \mathsf{F}(x+a)=\mathsf{F}(x)+b}} (-1)^{u^{\mathsf{T}}x+v^{\mathsf{T}}\mathsf{F}(x)}. \end{aligned} \tag{6}$$

For $u = v = 0$, Eq. (6) reduces to the probability of the differential with input difference a and output difference b, that is, $D^{\mathsf{F}}_{(0,b),(0,a)} = 2^{-n}\mathsf{DDT}^{\mathsf{F}}_{(a,b)}$. For $a = b = 0$, the coordinates of the correlation matrix of F can be obtained. In particularly, $D^{\mathsf{F}}_{(v,0),(u,0)} = C^{\mathsf{F}}_{v,u}$. Overall, the coordinates of D^{F} express the correlations of probabilistic linear relations ("linear approximations") between the input and output values of the right pairs.

Motivated by the notion of *linear trails*, Beyne and Rijmen propose the definition of *quasidifferential trails* in Definition 3, and show that exact expression for the probabilities of differentials can be given in terms of the correlations of quasidifferential trails in Theorem 1. For key-alternating ciphers, the expressions are shown in Theorem 2.

Definition 3 ([5]). *A quasidifferential trail for a function $\mathsf{F} = \mathsf{F}_r \circ \cdots \circ \mathsf{F}_1$ is a sequence $\omega_1, \ldots, \omega_{r+1}$ of mask-difference pairs $\omega_i = (u_i, a_i)$. The correlation of this quasidifferential trail is defined as $\prod_{i=1}^{r} D^{\mathsf{F}_i}_{\omega_{i+1},\omega_i}$.*

Theorem 1 ([5]). *Let $\mathsf{F} : \mathbb{F}_2^n \to \mathbb{F}_2^m$ be a function such that $\mathsf{F} = \mathsf{F}_r \circ \cdots \circ \mathsf{F}_1$. The probability of a characteristic with differences $a_1, \ldots, a_{r+1}$ is equal to the sum of the correlations of all quasidifferential trails with the same intermediate differences:*

$$\Pr[\wedge_{i=1}^{r}\mathsf{F}_i(x_i + a_i) = \mathsf{F}_i(x_i) + a_{i+1}] = \sum_{u_2,\cdots,u_r} \prod_{i=1}^{r} D^{\mathsf{F}_i}_{(u_{i+1},a_{i+1}),(u_i,a_i)}, \tag{7}$$

with $u_1 = u_{r+1} = 0$, $x_i = \mathsf{F}_{i-1}(x_{i-1})$ for $i = 2, \ldots, r$ and x_1 uniform random on $\mathbb{F}_2^n$.

Theorem 2 ([5]). *Let $\mathsf{F} = \mathsf{F}_r \circ \cdots \circ \mathsf{F}_1$ with $\mathsf{F}_i(x) = \mathsf{G}_i(x) + k_i$. If $\boldsymbol{k} = (\boldsymbol{k}_1, \ldots, \boldsymbol{k}_r)$ is a uniform random variable on a set $\mathcal{K}$, then*

$$\Pr[\mathsf{F}(x + a) = \mathsf{F}(x) + b] = \sum_{\substack{u_2,\ldots,u_r \\ a_2,\ldots,a_r \\ (u_2,\ldots,u_r)\perp\mathcal{K}}} \prod_{i=1}^{r} D^{\mathsf{G}_i}_{(u_{i+1},a_{i+1}),(u_i,a_i)}, \tag{8}$$

where $u_1 = u_{r+1} = 0$ and the probability is over a uniform random x and over the keys $\boldsymbol{k}_1, \ldots, \boldsymbol{k}_r$. In particular, for $\mathcal{K} = \mathbb{F}_2^n$, only quasidifferential trails with zero masks contribute to the key-averaged probability of the differential.

2.2 Description of GIFT Family

GIFT. To describe the attacks on GIFT, notations in Table 2 are used. Proposed by Banik et al. at CHES 2017 [4], GIFT has two versions: GIFT-64 and GIFT-128. GIFT adopts an SPN structure, with a 64-bit input for GIFT-64 and a 128-bit input for GIFT-128, both using a 128-bit key. The round numbers for GIFT-64 and GIFT-128 are 28 and 40, respectively. The function for each round is the same for both versions, composed of three operations: SubCells, PermBits, and AddRoundKey.

Table 2. Notations used in describing the attacks on GIFT.

X_i	the input state of the round i
$X_i[j]$	the j-th bit of the state X_i, $j = 0, \ldots, 127$, $X_i[127]$ is the most significant bit of X_i
$X_i[j \cdots k]$	the j-th bit, ..., k-th bit of the state X_i
$X_i[j_{n-1} \sim j_0]$	consecutive n bits from j_{n-1}-th to j_0-th bit of the state X_i
X_i^S	the state after the operation SubCells of round i
X_i^P	the state after the operation PermBits of round i
X_{i+1}	the state after the operation AddRoundKeys of round i
ΔX	the difference in state X
RK_i	the round key of the round i
$RK_i[j]$	the j-th bit of the i-th round key RK_i
k_i	the 16-bit word of the master key, $i = 0, \ldots 7$
k_i^j	the j-th bit of a 16-bit word k_i of the master-key
$\ggg$	circular right shift
P	the plaintext
C	the ciphertext
T	the tag of the output of encryption for SUNDAE-GIFT
RK_i'	is equal to $\texttt{PermBits}^{-1}(RK_i)$
$X_i'^P$	is equal to X_{i+1}

SubCells. Both versions of GIFT use the same invertible 4-bit S-box GS. The S-box is applied to every nibble of the internal state and is given in Appendix A, Table 9 of the full paper.

PermBits. The bit permutation maps bits from bit position i of the internal state to bit position $P(i)$: $b_{P(i)} \leftarrow b_i, i \in \{0, 1, \ldots, 63\}$ for GIFT-64, and $i \in \{0, 1, \ldots, 127\}$ for GIFT-128. Two tables of bit permutation used in GIFT-64 and GIFT-128 are given in Appendix A, Table 10 and Table 11 of the full paper, respectively.

AddRoundKey. For GIFT-64, the 32-bit round key $RK = U||V = u_{15} \cdots u_0||v_{15} \cdots v_0$, and is XORed with the internal state in the following way:

$$b_{4i+1} \leftarrow b_{4i+1} \oplus u_i,\ b_{4i} \leftarrow b_{4i} \oplus v_i,\ i \in \{0, 1, \cdots, 15\}.$$

For GIFT-128, the 64-bit round key $RK = U||V = u_{31} \cdots u_0||v_{31} \cdots v_0$, and is XORed with the internal state in the following way:

$$b_{4i+2} \leftarrow b_{4i+2} \oplus u_i,\ b_{4i+1} \leftarrow b_{4i+1} \oplus v_i,\ i \in \{0, 1, \ldots, 31\}.$$

AddRoundConstants. The round constants are given in Appendix A, Table 12 of the full paper.

Key Schedule. The 128-bit master key is initialized as $k_7||k_6|| \cdots ||k_1||k_0$, where k_i is 16-bit. For GIFT-64, two 16-bit words of the key state are extracted as the round key $RK = U||V$. $U \leftarrow k_1$, $V \leftarrow k_0$. For GIFT-128, four 16-bit words of the key state are extracted as the round key $RK = U||V$. $U \leftarrow k_5||k_4$, $V \leftarrow k_1||k_0$.

The key state is then updated as follows:

$$k_7||k_6 \cdots ||k_1||k_0 \leftarrow (k_1 \ggg 2)||(k_0 \ggg 12)|| \cdots ||k_3||k_2.$$

SUNDAE-GIFT. SUNDAE-GIFT is based on the mode of operation SUNDAE [1] at ToSC 2019, and the underlying block cipher is GIFT-128. The encryption algorithm takes as input an encryption key $K \in \{0,1\}^{128}$, an associated data $A \in \{0,1\}^*$, and a message $M \in \{0,1\}^{128}$. A nonce N with fixed length for variants is prepended on and regarded as a part of the associated data A. The output of the encryption is a ciphertext $C \in \{0,1\}^{|M|}$ and a tag $T \in \{0,1\}^{128}$. The operation "$\times$" denotes the multiplication by 2 or 4 depending on the length of the last blocks of A and M.

3 Revisiting and Discussion on the Weak-Key Conditions with Quasidifferential Approach [5]

This section provides a brief description of how to use the quasidifferential to derive the weak-key conditions for a *key-alternating cipher*, as given in [5]. We also discuss the assumptions implicitly used in this approach.

In the normal differential cryptanalysis, an r-round differential characteristic of a function $\mathsf{F} = \mathsf{F}_r \circ \cdots \circ \mathsf{F}_1$ with $\mathsf{F}_i = \mathsf{G}_i + k_i$ is a sequence

$$\varepsilon = \{a_1, a_2, \ldots, a_{r+1}\},$$

where a_i is the input differences of the i-th round. In the quasidifferential cryptanalysis, according to Beyne and Rijmen [5], an r-round quasidifferential trail corresponding to the above characteristic ε is the following sequence,

$$t = \{(u_1, a_1), (u_2, a_2), \ldots, (u_{r+1}, a_{r+1})\},$$

where (u_i, a_i) is the input mask-difference pair of the i-th round.

The correlation of a quasidifferential trail can be calculated by the quasidifferential transition matrix D^{G_i} of G_i, which is

$$cor = \prod_{i=1}^{r} (-1)^{u_{i+1}^{\mathsf{T}} k_i} D^{\mathsf{G}_i}_{(u_{i+1}, a_{i+1}), (u_i, a_i)}$$

Let $c = \prod_{i=1}^{r} D^{\mathsf{G}_i}_{(u_{i+1},a_{i+1}),(u_i,a_i)}$, we have

$$cor = (-1)^{\mathcal{K}} \cdot c = (-1)^{\mathcal{K}} \cdot (-1)^{b} \cdot |c|, \tag{9}$$

where $\mathcal{K} = \sum_{i=1}^{r} u_{i+1}^{\mathsf{T}} k_i$, $|c|$ is the absolute value of c, and b represents the sign of c, i.e., when $b = 1$, $c < 0$, otherwise, $c > 0$.

It is easy to check, the correlation $cor > 0$ under a condition $\mathcal{K} = b$, and $cor < 0$ under a condition $\mathcal{K} = b + 1$. According to Theorem 1, given a differential characteristic ε with *key-averaged probability*, denoted by p_{avg}, the exact probability of ε, denoted by p, is equal to the sum of correlations of all its corresponding quasidifferential trails. By Eq. (9),

$$p = \sum_i cor_i = \sum_i (-1)^{\mathcal{K}_i} \cdot c_i = \sum_i (-1)^{\mathcal{K}_i} \cdot (-1)^{b_i} \cdot |c_i|. \tag{10}$$

Unfortunately, the number of quasidifferential trails is too enormous to exhaust, thus, we actually cannot collect all quasidifferential trails, so we can only approximate the exact p by considering a fraction of the trails. For example, in [5], Beyne and Rijmen consider the quasidifferential trails with $|c| = p_{avg}$ for the analysis of `KNOT` and `RECTANGLE`, and $|c| = 2^0 \cdot p_{avg}$ to $|c| = 2^{-4} \cdot p_{avg}$ for the analysis of `SPECK`-32.

This paper follows a similar strategy in that we only consider those trails that have a significant contribution to the exact p. Divide all quasidifferential trails corresponding to ε into two parts according to $|c| = p_{avg}$ and $|c| < p_{avg}$, we have

$$p = p_{|c|=p_{avg}} + p_{|c|<p_{avg}} = \sum_{i,|c_i|=p_{avg}} (-1)^{\mathcal{K}_i} \cdot (-1)^{b_i} \cdot |c_i| + \sum_{j,|c_j|<p_{avg}} (-1)^{\mathcal{K}_j} \cdot (-1)^{b_j} \cdot |c_j|. \tag{11}$$

Assumption 1. *The probability of a differential characteristic ε whose average probability is p_{avg} is dominated by those quasidifferential trails with $|c| = p_{avg}$.*

Based on the Assumption 1, we approximate that

$$p \approx p_{|c|=p_{avg}} = \sum_{i,|c_i|=p_{avg}} (-1)^{\mathcal{K}_i} \cdot (-1)^{b_i} \cdot |c_i|. \tag{12}$$

Remark. Assumption 1 is intuitive. However, it is similar to the cases in the traditional differential and linear cryptanalysis, where dominating trails are used to approximate the real probabilities or correlations. For our applications in this paper, we have done experiments to verify Assumption 1 on `GIFT`-64 and `GIFT`-128 in Sect. 4.4, the experimental results are exactly consistent with our assumption, showing that Assumption 1 works well.

4 Derive Weak-Key Distinguishers of GIFT

In this section, we introduce how to apply the quasidifferential cryptanalysis and derive weak-key distinguishers for GIFT. In Sect. 3, we have revisited the roles of the keys in the exact probability of a differential characteristic. In our analysis of GIFT, we tend to use the differential (multiple differential characteristics sharing the same input and output differences) to amplify the probability. Therefore, the method in Sect. 3 cannot be trivially used as needs to handle more quasidifferential trails simultaneously. In Sect. 4.1, we introduce a method how to obtain the weak-key conditions for a differential and choose a good weak-key space. In Sect. 4.2 and 4.3, we obtain the weak-key distinguishers on GIFT-64 and GIFT-128, respectively. Finally, we verify Assumption 1 experimentally on GIFT-64 and GIFT-128 in Sect. 4.4, respectively, the experimental results are consistent with our assumption.

4.1 Obtain a Good Weak-Key Space for a Differential

According to Sect. 3, if a characteristic with p_{avg} has m quasidifferential trails satisfying $|c| = p_{avg}$, then $p \leq m \cdot |c|$ (under Assumption 1). The "=" case holds only if we can add m conditions such that

$$\mathcal{K}_i = b_i, \quad i = 1, \ldots, m.$$

Each condition above corresponds to an equation of keys, thus the m conditions lead to a weak-key space. Furthermore, there might be redundancy among the m conditions, i.e., the rank of the m conditions might be smaller than m, which we denote by ℓ. Thus, the size of this weak-key space is of size $2^{n-\ell}$ (n is the length of the key). When a key falls into this weak-key space, $p = m \cdot p_{avg}$.

As mentioned, we want to use the differential to enhance our attack. If a differential contains d differential characteristics, denoted by $\varepsilon_1, \ldots, \varepsilon_d$, respectively. According to Theorem 2 and Eq. (12), the probability p of this differential can be expressed as

$$p = \sum_{i=1}^{d} p_{\varepsilon_i} = \sum_{i=1}^{d} \sum_{\substack{j \\ |c_j^i| = p_{avg}^i}} (-1)^{\mathcal{K}_j^i} \cdot (-1)^{b_j^i} \cdot |c_j^i|. \tag{13}$$

For each characteristic ε_i above, suppose its average probability is p_{avg}^i, $i = 1, \ldots, d$. By applying ℓ_i equations to the keys, we can make the probability of ε_i be maximum, i.e., $m_i \cdot p_{avg}^i$. The corresponding weak-key space is denoted by W_i.

Not all W_i are compatible. A good weak-key space should satisfy two points: (a) the number of key conditions should be as small as possible; (b) the probability of the differential in this weak-key space should be as large as possible. To choose a good weak-key space, we choose the W_i with the maximum $\log_2(p) - \ell_i$. The algorithm procedure pseudo-code is shown in Appendix B, Algorithm 1 of the full paper.

4.2 Weak-Key Distinguishers for GIFT-64

In this subsection, we automate the search for quasidifferential trails following [5], and obtain weak-key conditions for distinguishers of GIFT-64 according to the discussion in Sect. 4.1. The probability of the 13-round differential of GIFT-64 in [9], which is used to launch the known best published 21-round differential attack on GIFT-64, is improved from $2^{-62.06}$ to $2^{-57.82}$ with 4-bit conditions of master-key, and is close to zero in some fraction of master-key. The probability of a 13-round multiple differentials of GIFT-64, which has the same input difference patterns and same output difference, is improved from $2^{-58.96}$ to $2^{-55.67}$ with 4-bit conditions of master-key. The details of the analysis are as follows.

The 13-Round Differential of GIFT-64. For the dominant characteristic with average probability $p_{avg} = 2^{-64}$, denoted by ε_1, of the 13-round differential of GIFT-64 presented in [9], we search and find 64 quasidifferential trails with $|c| = p_{avg}$ corresponding to ε_1. Take one of 64 trails, denoted by t_1, corresponding to ε_1 as an example. The ε_1 and t_1 are both listed in Table 3. The function for each round is $\mathsf{F}_i = \mathsf{G}_i + rk_i$ for $i = 1, \ldots, r$, where G_i is the PermBits $\circ$ SubCells operation, and rk_i is the AddRoundKey operation. According to Eq. (9), we have the correlation for t_1 is that

$$cor_1 = (-1)^{\sum_{i=1}^{r} u_{i+1}^{\mathsf{T}} rk_i} \cdot (-1)^{\sum_{i=1}^{r} b_i} \cdot \prod_{i=1}^{r} |D^{\mathsf{G}_i}_{(u_{i+1},a_{i+1}),(u_i,a_i)}| = (-1)^{\mathcal{K}_1} \cdot (-1)^0 \cdot 2^{-64}, \tag{14}$$

where $\mathcal{K}_1 = rk_1^0 + rk_1^{16} + rk_3^2 + rk_3^{17} + rk_5^0 + rk_5^{16} + rk_7^2 + rk_7^{17} + rk_9^0 + rk_9^{16} + rk_{11}^2 + rk_{11}^{17}$. Thus, a weak-key condition $\mathcal{K}_1 = 0$ for t_1 is obtained to ensure that $cor_1 > 0$. After performing a similar analysis for the other 63 trails, a total of 64 conditions are obtained.

It is easy to convert the 64 conditions about round-key bits into conditions about master-key bits as the linear key-schedule of GIFT. We get 4-bit conditions of the master-key after the Gaussian elimination of the 64 conditions. Suppose that four rounds are added before the 13-round distinguisher to launch the key-recovery attack, we get a weak-key space with 4-bit conditions of master-key, denoted by W_1:

$$k_0^4 + k_0^{12} = 0, k_4^5 + k_4^{13} = 0, k_0^0 + k_0^8 = 0, k_4^1 + k_4^9 = 0. \tag{15}$$

The size of the weak-key space W_1 is $2^{128-4} = 2^{124}$. In the weak-key space W_1, the probability of characteristic ε_1 is increased from 2^{-64} to $2^6 \cdot 2^{-64} = 2^{-58}$, and the probability of the differential 0x0000000000000202 $\xrightarrow{13\text{-}r}$ 0x0000000500000005 is improved from $2^{-62.06}$ to $2^{-57.82}$ after applying Algorithm 1.

The 13-round Multiple Differentials of GIFT-64. According to the 13-round multiple differentials in [9], we search for all 13-round characteristics with $p_{avg} \geq 2^{-64}$ satisfying the output difference 0x0000000000001010, and find 196 characteristics with $p_{avg} = 2^{-64}$. Furthermore, we find 33 of the 196 characteristics having the same input difference patterns and the same weak-key space,

Table 3. A quasidifferential trail t_1 corresponding to a differential characteristic ε_1.

r	a_r, SubCell(a_r) in ε_1, t_1	p^r_{avg}	u_r, SubCell(u_r) in t_1	$c_r = D^{\mathsf{G}_r}_{(u_{r+1},a_{r+1}),(u_r,a_r)}$	$(-1)^{u^{\mathsf{T}}_{r+1} rk_r}$	b_r
1	0x0000000000000202	2^{-4}	0x0000000000000000	$D^{\mathsf{G}_1}_{(5,5),(0,2)} D^{\mathsf{G}_1}_{(5,5),(0,2)} = 2^{-4}$	$(-1)^{rk_1^0 + rk_1^{16}}$	0
	0x0000000000000505		0x0000000000000505			
2	0x0000000500000005	2^{-6}	0x0000000500000005	$D^{\mathsf{G}_2}_{(0,2),(5,5)} D^{\mathsf{G}_2}_{(0,2),(5,5)} = 2^{-6}$		0
	0x0000000200000002		0x0000000000000000			
3	0x0000000002020000	2^{-4}	0x0000000000000000	$D^{\mathsf{G}_3}_{(5,5),(0,2)} D^{\mathsf{G}_3}_{(5,5),(0,2)} = 2^{-4}$	$(-1)^{rk_3^2 + rk_3^{17}}$	0
	0x0000000005050000		0x0000000005050000			
4	0x0000005000000050	2^{-6}	0x0000005000000050	$D^{\mathsf{G}_4}_{(0,2),(5,5)} D^{\mathsf{G}_4}_{(0,2),(5,5)} = 2^{-6}$		0
	0x0000002000000020		0x0000000000000000			
5	0x0000000000000202	2^{-4}	0x0000000000000000	$D^{\mathsf{G}_5}_{(5,5),(0,2)} D^{51}_{(5,5),(0,2)} = 2^{-4}$	$(-1)^{rk_5^0 + rk_5^{16}}$	0
	0x0000000000000505		0x0000000000000505			
6	0x0000000500000005	2^{-6}	0x0000000500000005	$D^{\mathsf{G}_6}_{(0,2),(5,5)} D^{\mathsf{G}_6}_{(0,2),(5,5)} = 2^{-6}$		0
	0x0000000200000002		0x0000000000000000			
7	0x0000000002020000	2^{-4}	0x0000000000000000	$D^{\mathsf{G}_7}_{(5,5),(0,2)} D^{\mathsf{G}_7}_{(5,5),(0,2)} = 2^{-4}$	$(-1)^{rk_7^2 + rk_7^{17}}$	0
	0x0000000005050000		0x0000000005050000			
8	0x0000005000000050	2^{-6}	0x0000005000000050	$D^{\mathsf{G}_8}_{(0,2),(5,5)} D^{\mathsf{G}_8}_{(0,2),(5,5)} = 2^{-6}$		0
	0x0000002000000020		0x0000000000000000			
9	0x0000000000000202	2^{-4}	0x0000000000000000	$D^{\mathsf{G}_9}_{(5,5),(0,2)} D^{\mathsf{G}_9}_{(5,5),(0,2)} = 2^{-4}$	$(-1)^{rk_9^0 + rk_9^{16}}$	0
	0x0000000000000505		0x0000000000000505			
10	0x0000000500000005	2^{-6}	0x0000000500000005	$D^{\mathsf{G}_{10}}_{(0,2),(5,5)} D^{\mathsf{G}_{10}}_{(0,2),(5,5)} = 2^{-6}$		0
	0x0000000200000002		0x0000000000000000			
11	0x0000000002020000	2^{-4}	0x0000000000000000	$D^{\mathsf{G}_{11}}_{(5,5),(0,2)} D^{\mathsf{G}_{11}}_{(5,5),(0,2)} = 2^{-4}$	$(-1)^{rk_{11}^2 + rk_{11}^{17}}$	0
	0x0000000005050000		0x0000000005050000			
12	0x0000005000000050	2^{-6}	0x0000005000000050	$D^{\mathsf{G}_{12}}_{(0,2),(5,5)} D^{\mathsf{G}_{12}}_{(0,2),(5,5)} = 2^{-6}$		0
	0x0000002000000020		0x0000000000000000			
13	0x0000000000000202	2^{-4}	0x0000000000000000	$D^{\mathsf{G}_{13}}_{(0,5),(0,2)} D^{\mathsf{G}_{13}}_{(0,5),(0,2)} = 2^{-4}$		0
	0x0000000000000505		0x0000000000000000			

which are listed in Appendix C, Table 13 of the full paper. Suppose that three rounds are added before the 13-round multiple differential distinguisher to launch the key-recovery attack, we get a weak-key space with 4-bit conditions of master-key, denoted by W_2:

$$k_2^1 + k_2^9 = 0, k_2^5 + k_2^{13} = 0, k_6^0 + k_6^8 = 0, k_6^4 + k_6^{12} = 0. \tag{16}$$

The size of W_2 is 2^{124}. In W_2, the probability of the 13-round multiple differentials, which contains 33 characteristics, is improved from $2^{-58.96}$ to $2^{-55.67}$.

4.3 Weak-Key Distinguishers for GIFT-128

Similarly, we find that for GIFT-128, the probability of the 20-round differential (presented in [25]) is improved from $2^{-121.83}$ to $2^{-114.77}$ with 6-bit conditions of

master-key, and the probability of a 21-round multiple differentials of GIFT-128 is improved from $2^{-128.38}$ to $2^{-122.77}$ with 4-bit conditions of master-key.

The 20-round Differentials of GIFT-128. For the 8 20-round differentials proposed in [25] (Table 7), which are used to launch the known best published 27-round differential attack on GIFT-128, a similar analysis is performed to obtain the weak-key spaces and probabilities. We find that the probability of differential 2 (0x000000000000000000000000000000a0 $\xrightarrow{20\text{-}r}$ 0x00000000000000002000000210000001), which contains 8 characteristics, is improved from $2^{-121.83}$ to $2^{-114.77}$ with 6-bit conditions of master-key. The size of the weak-key space is 2^{122}. The details are listed in Appendix D, Table 16 of the full paper. Suppose that four rounds are added before the 20-round distinguisher to launch the key-recovery attack, we get a weak-key space with 6-bit conditions of master-key, denoted by W_3:

$$k_0^9 + k_1^7 = 0, k_0^{11} + k_1^9 = 0, k_4^5 + k_5^1 = 0, k_6^9 + k_7^{11} = 0, k_6^{11} + k_7^{13} = 0, k_2^{15} = 1. \quad (17)$$

The 21-round Differentials of GIFT-128. We search for 21-round differentials that satisfy only one active S-box in the input difference, the output difference ΔOUT satisfies $\Delta OUT[127 \sim 64] = 0$ or $\Delta OUT[63 \sim 0] = 0$, and find 18 21-round differentials, which are listed in Appendix E, Table 19 of the full paper. Similarly, the quasidifferential trails for these differentials are searched, and the weak-key conditions for each differential are obtained. After that, we find 2 (differential 9 and 10 in Table 19 of the 18 differentials having the same input difference with the same 4-bit conditions of master-key. The details are listed in Appendix E, Table 20 of the full paper. Suppose that four rounds are added before the 21-round multiple differential distinguisher to launch the key-recovery attack, we get a weak-key space with 4-bit conditions of master-key, denoted by W_4:

$$k_4^4 + k_5^0 = 0, k_6^8 + k_7^{10} = 0, k_6^{10} + k_7^{12} = 0, k_2^1 = 1. \quad (18)$$

The size of W_4 is 2^{124}, and the probability of the 21-round multiple differentials, which contains 2 differentials, is improved from $2^{-128.38}$ to $2^{-122.77}$ in W_4.

4.4 Experiments

In Sect. 3 Assumption 1, we suppose that quasidifferential trails satisfying $|c| = p_{avg}$ are dominant for the probability of a characteristic. However, take the characteristic ε_1 in Table 3 as an example, when $\frac{|c|}{p_{avg}} \leq 2^{-6}$, a large number of quasidifferential trails exist. The number of quasidifferential trails of ε_1 from $\frac{|c|}{p_{avg}} = 2^0$ to $\frac{|c|}{p_{avg}} = 2^{-9}$ are listed in Table 4. To verify the effect of quasidifferential trails with $|c| < p_{avg}$ on the probability of characteristic can be ignored, we have done experimental verification on GIFT-64 and GIFT-128, respectively.

Experiments on GIFT-64. The 13-round characteristic ε_1 (listed in Table 3) is divided into two consecutive 4-rounds and one consecutive 5-round to experiment with the probability in the weak-key space W_1.

Table 4. The numbers of trails (denoted by # t) from $\frac{|c|}{p_{avg}} = 2^0$ to $\frac{|c|}{p_{avg}} = 2^{-9}$.

$-\log_2 \frac{\lvert c\rvert}{p_{avg}}$	0	1	2	3	4	5	6	7	8	9
# t	64	0	0	0	0	0	1024	0	1536	$\geq$ 5856

For each consecutive round, we conduct experiments under three cases: (1) round-key bits of each round are randomly generated, corresponding to the *key-averaged probability* for *Markov ciphers*; (2) the 128-bit master-key used in the key-schedule is randomly generated, corresponding to the key space of size 2^{128}; (3) the 128-bit master-key used in the key-schedule satisfies 4-bit conditions of W_1 (Eq. (15)), corresponding to the weak-key space of size 2^{124}.

In each case, 100 times are performed. Each time, plaintext pairs satisfying the input difference are randomly generated, the number of corresponding consecutive rounds is encrypted, and the number of right pairs (satisfying all intermediate differences) is counted. The experimental results are listed in Table 5. $pairs_{exp}$ represents the number of randomly generated plaintext pairs in each time. $pairs_{right}$ represents the number of right pairs counted by 100 times, and $p_{exp} = \frac{pairs_{right}}{pairs_{exp}}$.

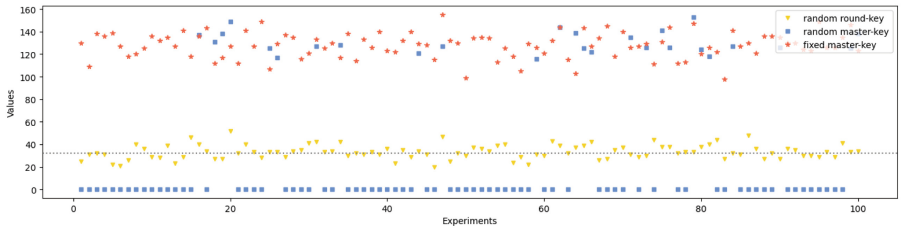
Table 5. Experimental results of GIFT-64.

round	p_{avg}	$pairs_{exp}$	Cases of key in the encryption					
			Random round-key		Random master-key		Fixed master-key	
			$pairs_{right}$	p_{exp}	$pairs_{right}$	p_{exp}	$pairs_{right}$	p_{exp}
1 to 4	2^{-20}	2^{25}	3325	$2^{-19.95}$	3385	$2^{-19.92}$	12786	$2^{-18.00}$
5 to 8	2^{-20}	2^{25}	3252	$2^{-19.98}$	3564	$2^{-19.84}$	12668	$2^{-18.01}$
9 to 13	2^{-24}	2^{29}	3204	$2^{-24.00}$	2842	$2^{-24.17}$	12805	$2^{-22.00}$

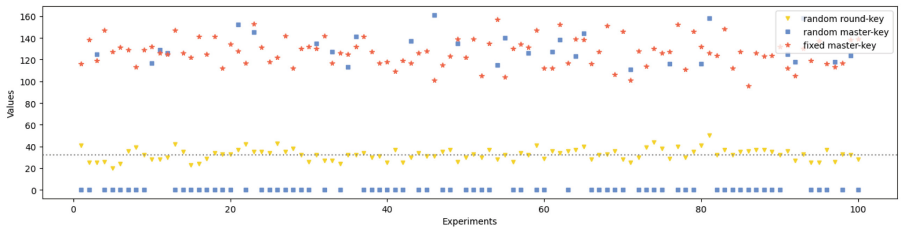
From the results in Table 5, the probabilities p_{exp} of case (3) in the weak-key space W_1 are much higher than case (2). The experimental probability of the characteristic ε_1 can be expressed as the product of two consecutive 4-rounds and one consecutive 5-round, i.e., $p \approx 2^{-58.01}$, which is close to 2^{-58} and exactly consistent with Assumption 1.

The numbers of right pairs of each experiment for three cases are shown in Fig. 1. In each subfigure, the lower triangles in yellow, the squares in blue, and the stars in red represent case (1), case (2), and case (3), respectively. It shows that the number of right pairs of case (2) is zero for some fraction of keys.

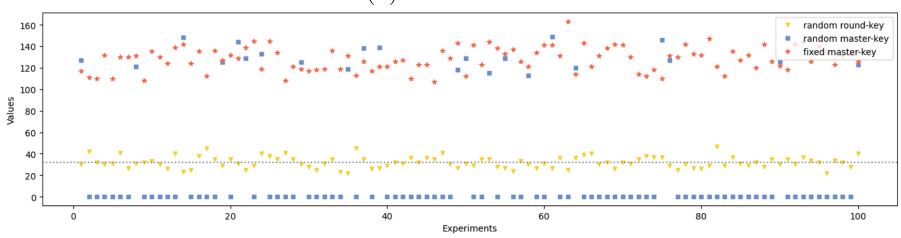
Experiments on GIFT-128. We also perform the experiments for six consecutive 2-rounds for the two dominant 21-round characteristics, which are listed in Appendix E, Table 21 of the full paper, to verify that the probability in the weak-key space W_4. The results are listed in Table 6, showing that Assumption 1 is reasonable.



(a) round 1 to 4



(b) round 5 to 8



(c) round 9 to 13

Fig. 1. Numbers of right pairs of 100 times for three cases of the key of GIFT-64.

5 Improved Weak-Key Key-Recovery Attacks on GIFT

In this section, we mount and reevaluate the attacks on GIFT-64, and GIFT-128, by the differentials in the weak-key spaces proposed in Sect. 4, respectively.

5.1 Weak-Key Key-Recovery Attacks on GIFT-64

The 21-round Differential Attack on GIFT-64. By adding four rounds before and four rounds after the 13-round differential distinguisher, which is presented in Sect. 4.2 with probability $2^{-57.82}$ in the weak-key space W_1, we launch a 21-round weak-key key-recovery attack on GIFT-64. The key-recovery process is shown in Fig. 2.

To facilitate representation and simplify the process, we perform an equivalent substitution for the AddRoundkey operation in the bottom of the distinguisher, and move it before the PermBits operation. The RK' is equals to the inverse of PermBits of RK, i.e., $RK' = \texttt{PermBits}^{-1}(RK)$. Additionally, accord-

Table 6. Experimental results of GIFT-128.

round	p_{avg}	$pairs_{exp}$	Cases of key in the encryption					
			Random round-key		Random master-key		Fixed master-key	
			$pairs_{right}$	p_{exp}	$pairs_{right}$	p_{exp}	$pairs_{right}$	p_{exp}
5 to 6	2^{-19}	2^{25}	6468	$2^{-18.99}$	7238	$2^{-18.81}$	12690	$2^{-18.00}$
8 to 9	2^{-9}	2^{15}	12854	$2^{-8.00}$	12698	$2^{-8.00}$	12696	$2^{-8.00}$
10 to 11	2^{-21}	2^{25}	1525	$2^{-21.07}$	1098	$2^{-21.54}$	6394	$2^{-19.00}$
13 to 14	2^{-9}	2^{15}	12704	$2^{-8.00}$	12916	$2^{-7.98}$	12608	$2^{-8.00}$
16 to 17	2^{-20}	2^{25}	3233	$2^{-19.98}$	3432	$2^{-19.90}$	6340	$2^{-19.01}$
19 to 20	2^{-9}	2^{15}	9577	$2^{-8.42}$	9578	$2^{-8.42}$	9687	$2^{-8.40}$

ing to the key schedule of GIFT-64, the round key RK_i and RK_j involve the same 32-bit master key when $i \equiv j \ mod \ 4$.

Data Collection. GIFT's structure allows us to freely extend one round because there is no whitening key at the beginning. Specifically, the structure can be constructed at X_1^P, while the plaintext P can be obtained by applying the inverse operation of PermBits (PermBits^{-1}) and SubCell (SubCell^{-1}) to X_1^P. Then encrypt the plaintext and obtain the corresponding ciphertext. By iterating all 64-bit in X_1^P, i.e., 2^{64} plaintexts, we can generate about $\binom{2^{64}}{2} \approx 2^{127}$ plaintext pairs.

Key Recovery Phase. Suppose that after data collection, we have 2^n plaintext pairs and corresponding ciphertext pairs. The corresponding bits of the master-key that need to be guessed during the key-recovery phase are listed in Table 7. The time complexity of each step with initial 2^n pairs is listed in Table 8. The detailed analysis of guessing keys and filtering to get the right pairs is given below. ? represents one bit of undetermined difference.

1). Guess 32-bit of RK_1: Guess 2-bit value of $RK_1[1,0]$, make the SubCell operation on the first S-box, remain the pairs satisfying

$$\texttt{SubCell}(X_1^P[3 \sim 0] \oplus RK_1[1,0]) \oplus \texttt{SubCell}(X_1^P[3 \sim 0] \oplus RK_1[1,0]) = 00?0,$$

and about $2^n \times 2^{-3}$ pairs left. Similarly, for the other 30-bit value of RK_1, i.e., $RK_1[31 \sim 2]$, guess each 2-bit RK_1 and perform a 3-bit filtering, around the right candidate pairs remain. The similar procedures are performed 16 times in total. Step 1) guesses 32-bit RK_1 in total, and about 2^{n-48} pairs left.

2). Guess 8-bit of RK_2: Guess 2-bit value of $RK_2[9,8]$, make SubCell on the 5-th S-box, remain the pairs satisfying
small

$$\texttt{SubCell}(X_2^P[18 \sim 15] \oplus RK_2[9,8]) \oplus \texttt{SubCell}(X_2^P[18 \sim 15] \oplus RK_2[9,8]) = 010?,$$

and about $2^{n-48} \times 2^{-3}$ pairs left. Guess $RK_2[13, 12]$, and perform a similar 3-bit filtering. Guess $RK_2[11, 10]$, make `SubCell` on the 6-th S-box, and discard the pairs that do not satisfy $\Delta X_3^S[23 \sim 20] = ?0?0$. Then a 2-bit filtering is performed. Similarly, guess $RK_{15,14}$ and perform a 2-bit filtering. Step 2) guess 8-bit RK_2 in total, and about 2^{n-58} pairs left.

3). Guess 4-bit RK_3: For each of the 2 active S-boxes in ΔX_3^P, guess the corresponding 2-bit RK_3, make `SubCell`, and perform a 3-bit filtering. Step 3) guesses 4-bit RK_3 in total, and about 2^{n-64} pairs left.

4). Guess 32-bit RK'_{21}: Note that all 32-bit subkeys of RK'_{21} are already guessed in Step 1), thus, for each of the 16 active S-boxes in $\Delta X_{21}^{'P}$, make $\texttt{SubCell}^{-1}$ on the corresponding S-box. Step 4) does not perform the filtering, and about 2^{n-64} pairs left.

5). Guess 32-bit RK'_{20}: Guess 2-bit $RK'_{20}[1, 0]$, make $\texttt{SubCell}^{-1}$, remain the pairs satisfying

$$\texttt{SubCell}^{-1}(X_{20}^{'P}[3 \sim 0] \oplus RK'_{20}[1, 0]) \oplus \texttt{SubCell}^{-1}(X_{20}^{'P}[3 \sim 0] \oplus RK'_{20}[1, 0]) = 0?0?,$$

and about $2^{n-64} \times 2^{-2}$ pairs left. For the other 15 active S-boxes in $\Delta X_{20}^{'P}$, 2-bit filtering is performed for each 2-bit RK'_{20} is guessed. The similar procedures are performed 16 times, and 32 bits RK'_{20} are guessed in Step 5) in total. After this step, there are about 2^{n-96} pairs left.

6). Guess 16-bit RK'_{19}: Similarly, for each one of 8 active S-boxes in $\Delta X_{19}^{'P}$, guess the corresponding 2-bit RK'_{19}, make $\texttt{SubCell}^{-1}$, and perform a 3-bit filtering. Step 6) guesses 16-bit RK'_{19}, and about 2^{n-120} pairs left.

7). Guess 4-bit RK'_{18}: Guess $RK'_{18}[1, 0]$ and $RK'_{18}[17, 16]$, make $\texttt{SubCell}^{-1}$, perform a 8-bit filtering in total. Step 7) guesses 4-bit RK'_{18}, and about 2^{n-128} pairs left.

Table 7. Involved keys in the 21-round differential attack on `GIFT`-64 of the 13-round differential. The keys in blue represent the bits that do not repeat the guess.

RK_1	$k_1^{15}k_0^{15}$	$k_1^{14}k_0^{14}$	$k_1^{13}k_0^{13}$	$k_1^{12}k_0^{12}$	$k_1^{11}k_0^{11}$	$k_1^{10}k_0^{10}$	$k_1^{9}k_0^{9}$	$k_1^{8}k_0^{8}$	$k_1^{7}k_0^{7}$	$k_1^{6}k_0^{6}$	$k_1^{5}k_0^{5}$	$k_1^{4}k_0^{4}$	$k_1^{3}k_0^{3}$	$k_1^{2}k_0^{2}$	$k_1^{1}k_0^{1}$	$k_1^{0}k_0^{0}$									
RK_2									$k_3^{7}k_2^{7}$	$k_3^{6}k_2^{6}$	$k_3^{5}k_2^{5}$	$k_3^{4}k_2^{4}$													
RK_3							$k_5^{9}k_4^{9}$								$k_5^{1}k_4^{1}$										
RK_4																									
RK'_{18}								$k_3^{14}k_2^{2}$								$k_3^{12}k_2^{0}$									
RK'_{19}		$k_5^{7}k_4^{11}$			$k_5^{15}k_4^{3}$			$k_5^{6}k_4^{10}$		$k_5^{14}k_4^{2}$		$k_5^{5}k_4^{9}$			$k_5^{13}k_4^{1}$			$k_5^{4}k_4^{8}$			$k_5^{12}k_4^{0}$			
RK'_{20}	$k_7^{3}k_6^{7}$	$k_7^{7}k_6^{11}$	$k_7^{11}k_6^{15}$	$k_7^{15}k_6^{3}$	$k_7^{2}k_6^{6}$	$k_7^{6}k_6^{10}$	$k_7^{10}k_6^{14}$	$k_7^{14}k_6^{2}$	$k_7^{1}k_6^{5}$	$k_7^{5}k_6^{9}$	$k_7^{39}k_6^{13}$	$k_7^{13}k_6^{1}$	$k_7^{0}k_6^{4}$	$k_7^{4}k_6^{8}$	$k_7^{8}k_6^{12}$	$k_7^{12}k_6^{0}$									
RK'_{21}	$k_1^{5}k_0^{3}$	$k_1^{9}k_0^{7}$	$k_1^{13}k_0^{11}$	$k_1^{1}k_0^{15}$	$k_1^{4}k_0^{2}$	$k_1^{8}k_0^{6}$	$k_1^{12}k_0^{10}$	$k_1^{0}k_0^{14}$	$k_1^{3}k_0^{1}$	$k_1^{7}k_0^{5}$	$k_1^{11}k_0^{9}$	$k_1^{15}k_0^{13}$	$k_1^{2}k_0^{0}$	$k_1^{6}k_0^{4}$	$k_1^{10}k_0^{8}$	$k_1^{14}k_0^{12}$									

Complexity and Success Probability. For the right key guesses, there are about $2^{n-64-57.82}$ pairs left, while for the wrong key guesses, about 2^{n-128} pairs left. We set $n = 121.82$ to ensure that at least one pair is remained for the right key guesses, while about $2^{-6.18}$ pairs are remained for the wrong key guesses. Therefore, the data complexity is about 2^{64} chosen-plaintexts, the time complexity is

Table 8. Time complexity of the 21-round differential attack on GIFT-64 in each step.

Step	RK	# Key	Time(S-box operations)	Filtering probability	# Remaining pairs
1.	RK_1	2^{32}	$2 \times 2^n \times 2^3$	$2^{-3\times16}$	2^{n-48}
2.	RK_2	2^8	$2 \times 2^{32} \times 2^{n-48} \times 2^3$	$2^{-3\times2-2\times2}$	2^{n-58}
3.	RK_3	2^4	$2 \times 2^{40} \times 2^{n-58} \times 2^3$	$2^{-3\times2}$	2^{n-64}
4.	RK'_{21}	-	$2 \times 2^{44} \times 2^{n-64} \times 16$	-	2^{n-64}
5.	RK'_{20}	2^{32}	$2 \times 2^{44} \times 2^{n-64} \times 2^6$	$2^{-2\times16}$	2^{n-96}
6.	RK'_{19}	2^{16}	$2 \times 2^{76} \times 2^{n-96} \times 2^3$	$2^{-3\times8}$	2^{n-120}
7.	RK'_{18}	2^4	$2 \times 2^{92} \times 2^{n-120} \times 2^{2.32}$	$2^{-4\times2}$	2^{n-128}

dominated by Step 1), and about $2^{125.82} \cdot \frac{1}{16} \cdot \frac{1}{21} \approx 2^{117.42}$ 21-round operations, the memory complexity is about 2^{96}-bit. We use the formula presented by Selçuk in [17] to evaluate the success probability P_S:

$$P_S = \Phi\left(\frac{\sqrt{\mu S_N} - \Phi^{-1}(1-2^{-a})}{\sqrt{S_N+1}}\right), \tag{19}$$

where p is the probability of the differential, N is the plaintext-ciphertext pairs, $\mu = pN$, S_N is the signal-to-noise, and a is an a-bit or higher advantage. We have $S_N = 2^{6.18}$. For $n = 121.82$, $\mu = 1$, $a = 5$, $P_S \approx 81.06\%$. For $n = 125$, $\mu = 2^{3.18}$, $a = 15$, $P_S \approx 99.41\%$, and time complexity is about $2^{120.6}$ 21-round operations.

Remark. The calculation of the time complexity of the 21-round differential attack proposed in [9], which is the combination of the 1-round attack and the 20-round, is not accurate, we reevaluate the time complexity utilizing their 13-round differential with average probability $2^{-62.06}$ based on the above analysis. Then $S_N = \frac{2^{n-64-62.06}}{2^{n-128}} = 2^{1.94}$. For $n = 126.06$ to ensure that at least one right pair left, $\mu = 1$, $P_S \approx 51.60\%$ for $a = 5$, and the time complexity is about $2^{121.66}$ 21-round operations. Our attack improves the success probability and complexity in the weak-key space W_1 compared to [9].

The 21-round Multiple Differential Attack on GIFT-64. Based on the 13-round multiple differentials proposed in Sect. 4.2 with probability $2^{-55.67}$ in the weak-key space W_2, we add three rounds before and five rounds after the distinguisher to mount the 21-round multiple differential attack on GIFT-64 in Appendix C of the full paper. Thanks to increasing the probability of the 13-round multiple differential distinguisher in the weak-key space W_2, it allows us to boost the multiple differential attack on GIFT-64 from 20 to 21 rounds.

Complexity and Success Probability. The data complexity is 2^{64} chosen-plaintexts, the time complexity is about $2^{123.27}$ 21-round operations, the memory complexity is about 2^{112}-bit, and the success probability is about 99.9%.

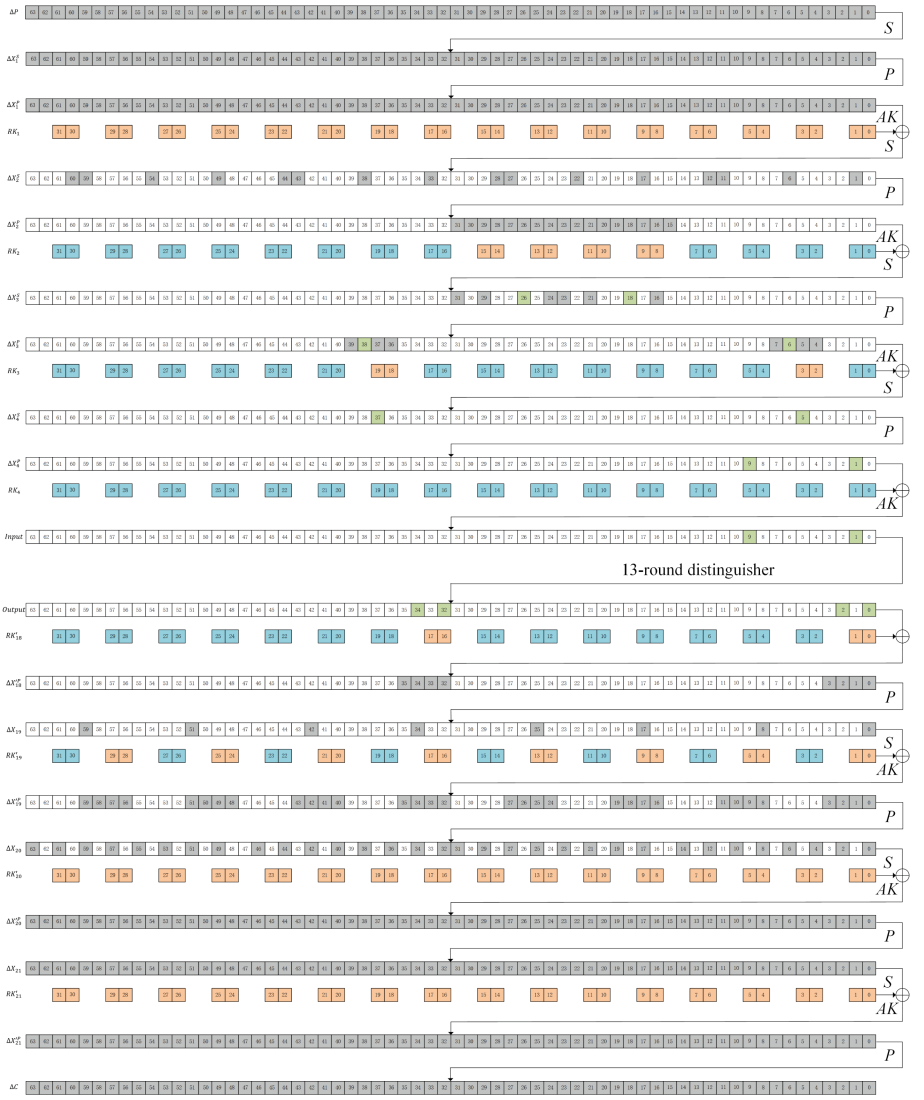


Fig. 2. 21-Round differential attack on GIFT-64. Each square represents one bit. The squares in white stand for the values of difference bits are '0'. The squares in green stand for the values of difference bits are '1'. The squares in grey stand for the values of difference bits that are indeterminate. The squares in orange stand for the bits of key are need to be guessed. The squares in blue stand for the bits of key that are omitted. (Color figure online)

5.2 Weak-Key Key Recovery Attacks on GIFT-128

The 27-round Differential Attack on GIFT-128. Using the 20-round differential with probability $2^{-114.77}$ in the weak-key space W_3, which is presented in

Sect. 4.3, we launch the differential attack on 27-round GIFT-128 by adding four rounds before and three rounds after the distinguisher in Appendix D of the full paper.

Complexity and Success Probability. The data complexity is about $2^{115.77}$ chosen-plaintexts, the time complexity is about $2^{110.85}$ 27-round operations, the memory complexity is about 2^{92}-bit, and the success probability is about 99.9%. Compared to the 27-round differential attack in [25], the complexity is reduced by about $2^{3.06}$ in a weak-key setting.

The 28-round Differential Attack on GIFT-128. Similarly, by adding four rounds before and three rounds after, the first 28-round multiple differential attack on GIFT-128 is launched based on the 21-round multiple differentials with probability $2^{-122.77}$ in the weak-key space W_4, which is proposed in Sect. 4.3. The details of the attack are presented in Appendix E of the full paper.

Complexity and Success Probability. The data complexity is about $2^{123.77}$ chosen-plaintexts, the time complexity is about $2^{123.77}$ 28-round operations, the memory complexity is about 2^{96}-bit, and the success probability is about 86.5%.

6 Distinguisher and Attack on SUNDAE-GIFT

We attack the initialization phase without plaintext data of version SUNDAE-GIFT-96 of SUNDAE-GIFT family with a 96-bit nonce, which is the primary member satisfying the requirements set by NIST. Our restriction for searching distinguishers is that the difference of the plaintext is only active in the high 96 bits by adding a certain number of rounds before the distinguisher. Then we find two 11-round differential characteristics, which are listed in Appendix F, Table 25 of the full paper with average probability 2^{-60}, that can be utilized to launch a 17-round differential attack by adding three rounds before and three rounds after the distinguisher. Similarly, we search for quasidifferential trails satisfying $|c| = p_{avg}$ and find only one quasidifferential trail with all-zero masks for each characteristic, thus, there are no weak-key conditions for these two characteristics.

The first 17-round differential attack on SUNDAE-GIFT utilizing the 11-round differential characteristic with probability 2^{-60} is proposed in Appendix F of the full paper.

Complexity and Success Probability : The data complexity is about $2^{62.5}$ chosen-plaintexts, the time complexity is about $2^{75.37}$ 17-round operations, the memory complexity is about 2^{66}-bit, and the success probability is about 99.9%.

7 Conclusion

In this paper, we continue the work on the quasidifferential transition matrix in differential cryptanalysis proposed by Beyne and Rijmen at CRYPTO 2022 and

apply their approach to GIFT. By holding some conditions of master-key, the probabilities of some differential characteristics with small probabilities can be improved in a weak-key setting. Then the weak-key (multiple) differential key-recovery attacks on GIFT are obtained based on the weak-key distinguishers. For GIFT-64, the multiple differential attack can be boosted from 20 to 21 rounds, and the 21-round differential attack can be improved in terms of complexity. For GIFT-128, the complexity of the 27-round differential attack can be improved, and the first 28-round (multiple) differential attack is obtained. Finally, after checking the 11-round differential characteristic with quasidifferential trails, we mount the first differential attack on the 17-round SUNDAE-GIFT.

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